Supersymmetric Rényi Entropy

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(U. Tokyo)

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Introduction

Entanglement
Entropy

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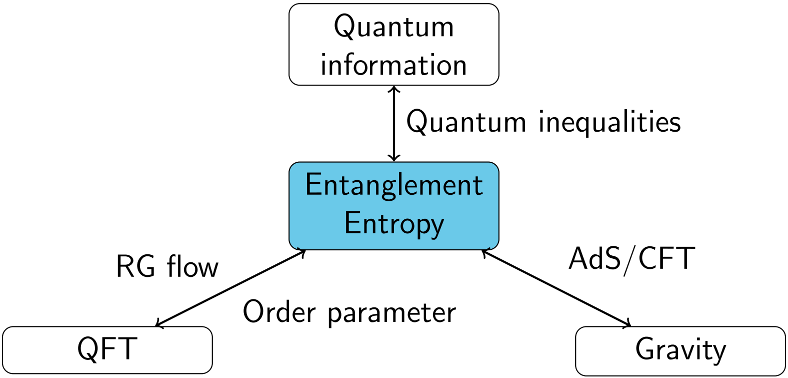

What's the role of entanglement entropy in QFT?

- Entanglement entropy as a measure of degrees of freedom

- Construct a monotonic function $c(\text{Energy})$ of the energy scale
  - Entropic c-theorem in two dimensions
  - F-theorem in three dimensions

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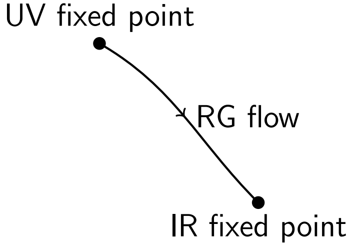

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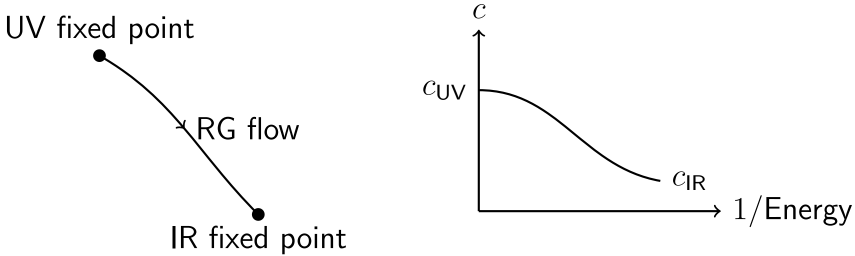

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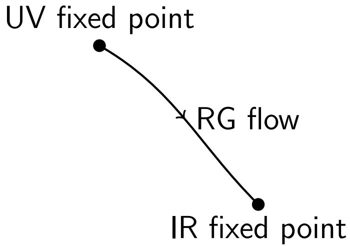

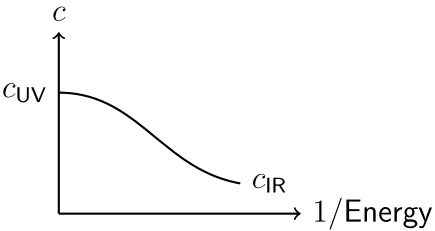

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  - Similarity between MERA and AdS space
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Questions to be addressed

- If no, what is the most natural extension of them?
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Outline

1. Review of entanglement entropy

2. Entanglement entropy and RG flow
   - C-theorem in 3d?
   - F-theorem

3. Supersymmetric Rényi entropy
   - 3d $\mathcal{N} = 2$ theories
   - Holographic supersymmetric Rényi entropy
   - 5d $\mathcal{N} = 1$ theories

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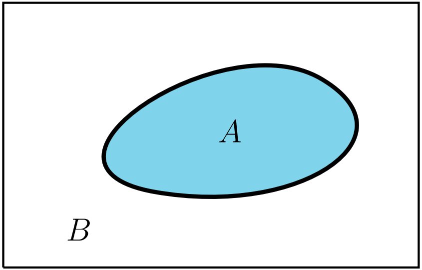
Definition of entanglement entropy

- Divide a system to A and $B = \bar{A}$: $\mathcal{H}_{tot} = \mathcal{H}_A \otimes \mathcal{H}_B$



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$$S_A = -\mathrm{tr}_A \rho_A \log \rho_A$$

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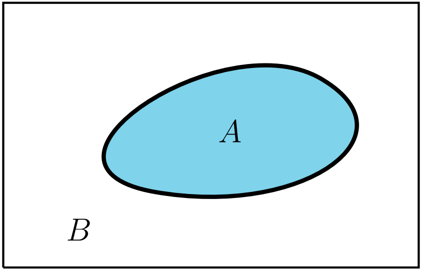

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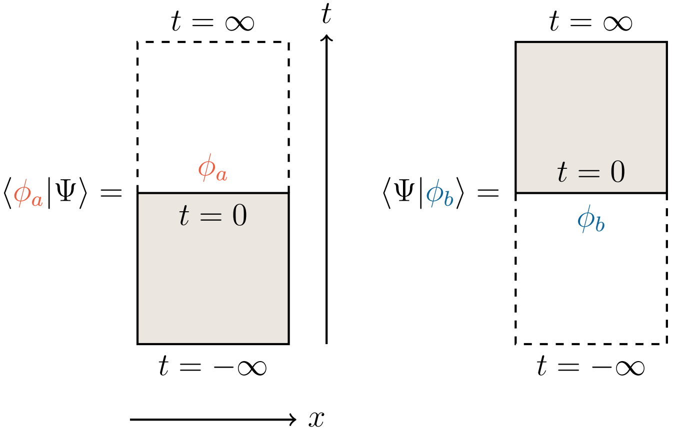
- Reduced density matrix:

$$\rho_A = \mathrm{tr}_B \rho_{tot} = \sum_i \langle \psi_B^i | \rho_{tot} | \psi_B^i \rangle$$

$\mathcal{H}_B = \{|\psi_B^1\rangle, |\psi_B^2\rangle, \cdots\}$ orthonormal basis

Path integral representation of the wave function



States $|\phi_{a,b}\rangle$ are the boundary conditions at $t = 0$

Replica trick and covering space

$$[\rho_A]_{ab} = \frac{1}{Z_1} \int [\mathcal{D}\phi^B(t=0, \vec{x} \in B)] \left(\langle \phi_a^A | \langle \phi^B | \right) |\Psi\rangle\langle\Psi| \left(|\phi_b^A\rangle |\phi^B\rangle \right) ,$$

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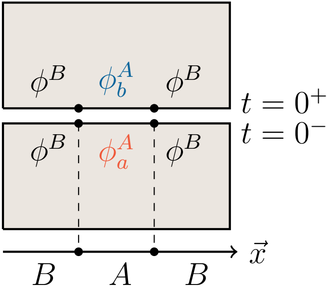

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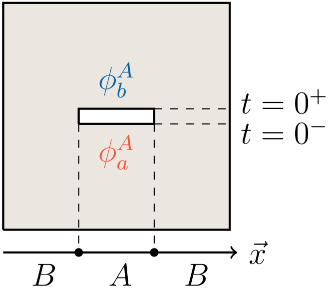
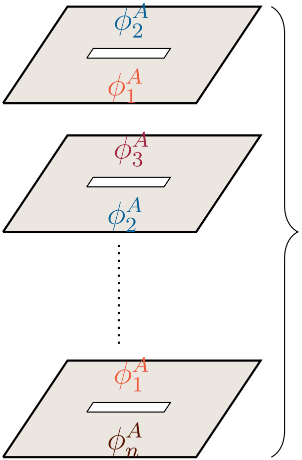
$$[\rho_A]_{ab} = \frac{1}{Z_1}$$



Replica trick and covering space

$$\mathrm{tr}_A \rho_A^n = \frac{1}{(Z_1)^n}$$



n copies $\equiv Z_n$

$$= \frac{Z_n}{(Z_1)^n}$$

Replica trick and covering space

Entanglement entropy

$$S_A = -\left(\partial_n - 1\right) \log Z_n \Big|_{n=1}$$

All we need to know is the partition function Z_n on the n-fold cover $\mathcal{M}_n$!

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One parameter extension ($S_A = \lim_{n\to 1} S_n$)

n-th Rényi entropy

$$S_n = \frac{1}{1-n} \log \frac{Z_n}{(Z_1)^n}$$

Properties of entanglement entropy

For a pure ground state

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Strong subadditivity [Lieb-Ruskai 73]

$$S_{A\cup B\cup C} + S_B \le S_{A\cup B} + S_{B\cup C}$$
$$S_A + S_C \le S_{A\cup B} + S_{B\cup C}$$

for any three disjoint regions A, B and C

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$$F_{\text{Therm}} \sim c_{\text{Therm}} T^3$$

C_T-theorem [Petkou 94]

$$C_T|_{\text{UV}} \geq C_T|_{\text{IR}} \ , \qquad \langle T_{\mu\nu}(x) T_{\rho\sigma}(0) \rangle_{\text{CFT}} = C_T \frac{I_{\mu\nu,\rho\sigma}(x)}{x^6}$$

F-theorem [Jafferis-Klebanov-Pufu-Safdi 11, Myers-Sinha 10]

$$F_{\text{UV}}(\mathbb{S}^3) \geq F_{\text{IR}}(\mathbb{S}^3) \ , \quad F = -\log Z(\mathbb{S}^3)$$

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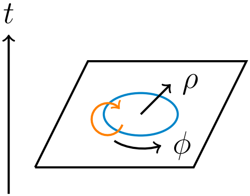

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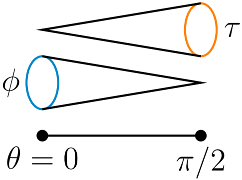

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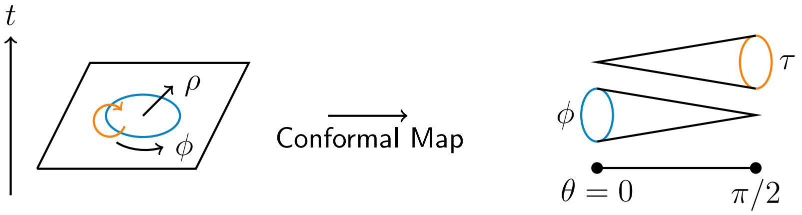

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$\mathbb{S}^3_n$: n-fold cover of $\mathbb{S}^3$

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Taking $n \to 1$ limit

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Proof of the F-theorem by using entanglement entropy?

Renormalized entanglement entropy

Two requirements for the proof of the F-theorem

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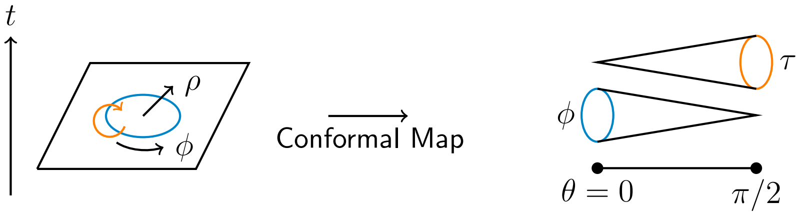

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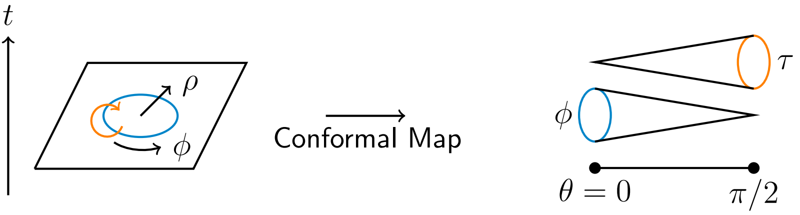

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- For SUSY gauge theories, $Z[\mathbb{S}^3]$ $(n = 1)$ can be obtained by localization [Kapstin-Willet-Yaakov 09, Jafferis, Hama-Hosomichi-Lee 10]

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- SUSY is broken on the singular space $\mathbb{S}^3_n$

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SUSY partition function on $\mathbb{S}_n^3$

$$Z^{\text{susy}}[\mathbb{S}_n^3] = Z^{\text{susy}}[\mathbb{S}_b^3]$$

$\mathbb{S}_b^3$: squashed three-sphere with squashing parameter $b = \sqrt{n}$

Some properties

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Expansion around $n = 1$

$$S_n^{\text{susy}} = S_1 + \frac{\pi^2}{16}\tau_{rr}(n-1) + \cdots$$

$$\left(\langle j_\mu^{(R)}(x) j_\nu^{(R)}(0)\rangle \sim \tau_{rr} I_{\mu\nu}(x)\right)$$

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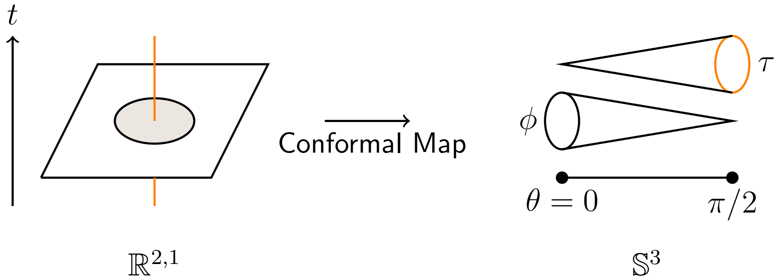
Large-N limit

$$S_n^{\rm susy} = \frac{3n+1}{4n} S_1$$

Adding Wilson loop

- The quark insertion is equivalent to the Wilson loop [Lewkowycz-Maldacena 13]



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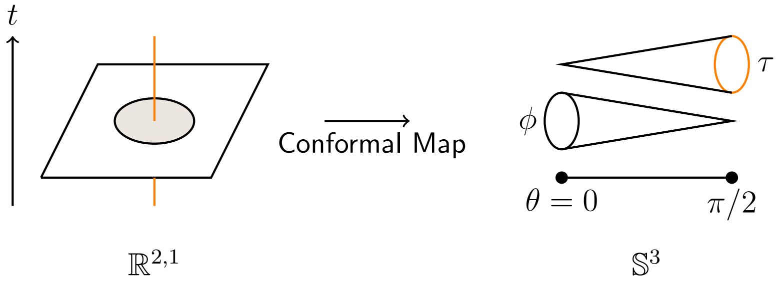

The variation of RE by the loop

$$S_{W,n} = \frac{1}{1-n}\left(n \log |\langle W \rangle_1| - \log |\langle W \rangle_n|\right)$$

Wilson loop in the large-N [TN 14]

- $1/6$-BPS Wilson loop in ABJM on $\mathbb{S}^3_n$

$$\log\langle W\rangle_n = \frac{\pi(n+1)}{2}\sqrt{2\lambda} + O(\log N)$$

λ: 't Hooft coupling $\lambda \equiv N/k$

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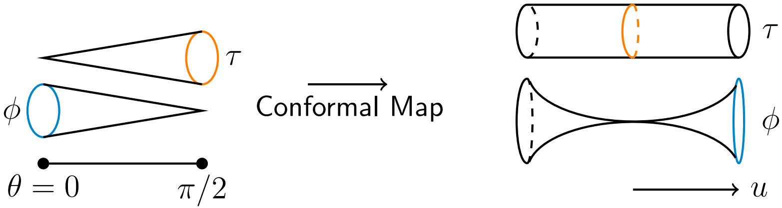
- The variation does not depend on the Rényi parameter n!

The SRE of the Wilson loop

$$S^{\text{susy}}_{W,n} = \frac{\pi}{2}\sqrt{2\lambda}$$

Further conformal map



$\mathbb{S}^3$ $\quad\quad$ $\mathbb{S}^1 \times \mathbb{H}^2$

$$ds^2 = d\theta^2 + \sin^2\theta d\tau^2 + \cos^2\theta d\phi^2$$

$$ds^2 = d\tau^2 + du^2 + \sinh^2 u d\phi^2$$

The n-fold cover has $\tau \sim \tau + 2\pi n$

The dual gravity solution

- The $1/2$-BPS $U(1)$ charged topological AdS$_4$ black hole in $\mathcal{N}=2$ gauged SUGRA

$$ds^2 = f(r)d\tau^2 + \frac{dr^2}{f(r)} + r^2 ds^2_{\mathbb{H}^2}$$
$$A = Q\left(\frac{1}{r} - \frac{1}{r_H}\right) d\tau$$

The horizon is at $r = r_H$ where $f(r_H) = 0$

- The temperature:

$$T = \frac{2r_H - 1}{2\pi}$$

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Holographic supersymmetric Rényi entropy

- Choose the temperature to be $T = \frac{1}{2\pi n}$
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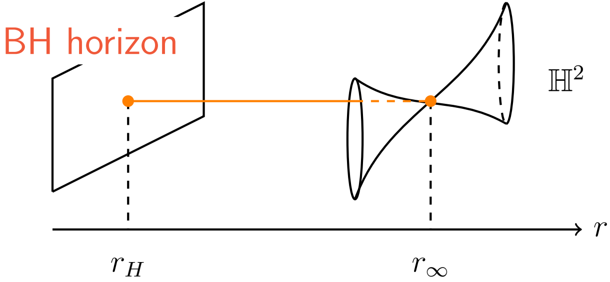
Agrees with the large-N result! [Huang-Rey-Zhou 14, TN 14]

Adding holographic Wilson loop

- The fundamental string dual to the Wilson loop



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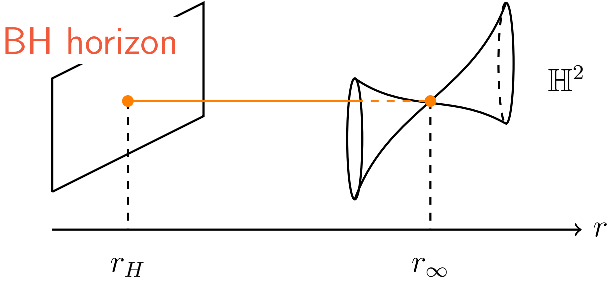

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n-fold cover of $\mathbb{S}^5$ [Hama-TN-Ugajin 14]

- Consider $\mathcal{N} = 1$ five-dimensional theories on $\mathbb{S}^5_n$

$$ds^2 = d\theta^2 + n^2 \sin^2 \theta d\tau^2 + \cos^2 \theta ds^2_{\mathbb{S}^3}$$

- The background $SU(2)_R$ gauge field $V^I{}_J$ $(I, J = 1, 2)$ for SUSY

$$V^1{}_1 = -V^2{}_2 = \pm i \frac{n}{2} d\tau$$

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The localized partition function

$$Z^{\text{susy}}[\mathbb{S}^5_n] = Z^{\text{susy}}[\mathbb{S}^5_{(\omega_1,\omega_2,\omega_3)=(1/n,1,1)}]$$

$\mathbb{S}^5_{(\omega_1,\omega_2,\omega_3)}$: squashed sphere

5d SRE in the large-N limit

- Consider $\mathcal{N}=1$ $USp(2N)$ gauge theories with N_f flavors and a single antisymmetric hypermultiplet

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Large-N limit

$$S_n^{\mathrm{susy}} = \frac{19n^2 + 7n + 1}{27n^2} S_1$$

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With a BPS Wilson loop

$$S_{W,n}^{\text{susy}} = \sqrt{\frac{2}{8 - N_f}} \pi N^{1/2}$$

Holographic SRE in 5d

- The BPS charged topological AdS_6 black hole in the Romans $F(4)$ SUGRA asymptotes to $\mathbb{S}^1 \times \mathbb{H}^4$
- The holographic SRE agrees with the large-N result!
  [Alday-Richmond-Sparks 14], [Hama-TN-Ugajin 14]
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Summary

- SUSY does not manifest itself in the Rényi entropy
- A new observable, supersymmetric Rényi entropy, is introduced
- The holographic duals of the supersymmetric Rényi entropies are given by the BPS charged topological AdS black holes

Future direction

- Can SRE be defined in other dimensions? (4d SRE by [Huang-Zhou 14], [Crossley-Dyer-Sonner 14])
- Boundary SRE or squashed SRE?
- Entangling surface as a surface operator? [Drukker-Okuda-Passerini 14]